COSMIC CYCLES AND METHODOLOGICAL TRIANGLE
Milanković’s unscrambling of Pleistocene ice ages

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Serbian mathematician Milutin Milanković is considered to be the founder of the modern astronomical theory of climate change. In 1912, in an article "On the Mathematical Theory of Climate," Milanković introduced, for the first time, advanced mathematics into climatology. In this and subsequent publications, he demonstrated the interrelatedness of celestial mechanics and the earth sciences, thus inaugurating the transformation of climatology and related descriptive sciences into exact ones.

Key words: Milutin Milanković, Serbian science, climatology, interdisciplinarity

Contemporary climatology is based upon works of Milutin Milanković (1879 – 1958), professor of Applied Mathematics at Belgrade University. He enabled consistent transition from celestial mechanics to the earth sciences and transformation of descriptive sciences into exact ones. Considering seasonal and latitudinal distribution of earth’s insolation, caused by changes in earth’s orbital geometry, Milanković formulated the only theory of climate that can be verified mathematically and tested geologically. In six papers published from 1912 to 1914 he formulated a precise, numerical climatological model with capacity for reconstruction of the past and prospecting of the future, and established the astronomical theory of climate as a generalized mathematical theory of insolation.

Milutin Milanković revolutionized understanding of climate dynamics, set up climatology as an exact science and initiated numerical modeling of the climate. He founded cosmic climatology by calculating temperatures of the upper layers of the Earth's atmosphere as well as the temperature conditions on planets of the inner Solar system, Mercury, Venus, Mars and the Moon, and the depth of the atmosphere of the outer planets — results that were mainly supported by later observations. His concept opened cosmic perspective of climatology, facilitated mathematical interpretation of long quasi-periodic climate changes, and became an organon for the understanding of earth’s ice ages which is one of the main scientific challenges today.

In particular he calculated the impact of the Earth’s secular orbital cycles on climate changes and explained the origin of the Pleistocene ice ages. The perennial periodic orbital variations considered in his Canon of Insolation (eccentricity, obliquity, precession), along with their influence on planets’ climates, today are called Milanković cycles. His theory, which provided mathematical and physical basis for observationally well established cyclic nature of climatic variations, is the most significant work of the Belgrade School of Climatology and Meteorology, which was initiated in 1848 by the work of Vladimir Jakšić. Today it continues to be
recognized world-wide in weather modeling and forecasting (The Eta-model developed by Mesinger and Janjić).\(^1\)

The idea of possible astronomically related climate changes emerged in the European scientific community almost two centuries ago. First it was considered by astronomers (Herschel) and than postulated by geologists (Agassis). After that, it took a few decades for earth sciences to assure itself, by geological data, in the existence of several "ice ages" which froze vast areas of Eurasia and Northern America significantly lowering the level of the oceans and seas. As soon as the idea became plausible, the earth scientists started making hypothesis to explain it. Many trials had been made: some almost immediately proved to be wrong, some were partially successful and lasted for some time, some were completely undisputable, suitable neither to be proved, nor to be opposed. The most of those hypothesis were dealing with processes observed (or supposed to be possible) on the Earth: high content of volcanic dust in the atmosphere, perturbations of the Earth's magnetic field, fluctuations in the distribution of carbon dioxide between atmosphere and the ocean, changes of the deep circulation of the ocean...

Parallely, there were also several attempts to explain the climate change by the influence of astronomical forces (the most comprehensive of them was the theory by James Croll in '60-ties of XIX century). But, due to many scientific, personal and social circumstances, none of them could prove to be accurate. Moreover, the imperfection of such theories was understood as inaptitude of any astronomical factor to be the cause of climatic change. Therefore the riddle of ice ages remained unsolved for another several decades.

In XX century, between the two world wars, the solution finally emerged through the work of Milutin Milanković, professor of the Applied Mathematics in the Belgrade University.\(^2\) He was born in village of Dalj, in Slavonia, part of the Austria-Hungary, where his ancestors settled at the end of XVII Century after the great migration of Serbs from Kosovo and Metohija province escaping the Turkish yoke. His wealthy family, through the centuries esteemed by philosophers, inventors, professors, lawyers, civil servants, was enthusiastic to support the education of its children. As a schoolboy, Milutin had an excellent high school teacher of mathematics (Vladimir Varicak, later an academician) who noticed his exceptional abilities and directed him toward mathematical science. "Profesor Varicak", said Milanković in his memoirs, "considered each of my errors in calculations as a sign of character weakness and said that he would not give me the mark 'excellent' until I became flawless".\(^3\) Later, he enrolled in the Technical School in Vienna, with major in Civil engineering, and finally, in 1904, was awarded a Ph. D. in technical sciences. After a relatively short but very successful career as an engineer, Milanković abandoned Vienna and, in 1909, on an invitation of the Ministry of Education of the Kingdom of Serbia, took the position of a professor in the College of Philosophy in Belgrade.

Milanković's position at the University immersed him in a non-specialized, holistic culture of university education in Serbia at the beginning of 20th century and allowed him to work at the intersection of a number of scientific fields. It provided him with a broad vision of an integrated cosmic science that could be applied to very specific problems in geophysics, climatology - including the problem of past ice ages, and the temperatures of other planets. University blueprint, "acquiring unity among the sciences," was in complete dissonance with the

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prevailing scientific specialization-driven culture of Europe of that time, but without that cultural
peculiarity the problems he correctly solved would not even have been posed. Milanković himself
was assured that the mystery of the ice ages had not been resolved yet because its solution was
hidden somewhere in the interstices between many individual disciplines. He wrote in his
memoirs:

“The reason for that lays in the fact that one has, in order to get to the bottom of the
problem, to solve a set of rather complicated component problems which really belong to different
sciences that are sharply separated one from the other... Therefore, the question was not answered,
and it was left amid a triangle between spherical astronomy, celestial mechanics and theoretical
physics. The chair at Belgrade University offered to me included all the three sciences which were
separated at other Universities. Therefore I was able to discern that cosmic problem, to see his
importance and to start with its unraveling.”

This methodological approach, substantially different from other universities, forced him
to set appropriate, firm ground for understanding the core of the climate problem.

“That coincidence, which enabled me to adhere to the given problem, it was not pure
accident although it looked like that. Exactly because I was involved in all the mentioned sciences,
it was possible for me to smell out that problem and to estimate its importance.”

He was aware that divided scientific experience, separated astronomical and geological
thinking, make the comprehension of the climate impossible. It was a blind way which ultimately
should be replaced by a new, open methodology - the triangle which couples sciences to frame
Proteus like climate dynamics. Therefore Milanković’s climatology was one among the most
powerful and productive apologies of interdisciplinarity at the beginning of the XX century. He
began working on it in 1912, after he had realized that "most of meteorology is nothing but a
collection off innumerable empirical findings, mainly numerical data, with traces of physics used
to explain some of them... Mathematics was even less applied, nothing more than elementary
calculus... Advanced mathematics had no role in that science...”

The work on the problem of ever changing climate begun by dealing with notable
previous theories, with determining their lacks and faults. Milanković tried to continue the work of
his predecessors, but in a basically different way: he was not searching for the causes of the Earth's
ice ages, but trying to develop a general mathematical theory of climate applicable to all planets.
His aim was an integral, mathematically accurate theory which connects thermal regimes of the
planets to their movement around the Sun. He wrote: "...such a theory would enable us to go
beyond the range of direct observations, not only in space, but also in time...” It would allow
reconstruction of the Earth's climate, and also its predictions, as well as give us the first reliable
data about the climate conditions on other planets.

In the center of his theory Milanković put the Sun, the only source of heat and light in
the Solar system, the Earth being only a part of it. The planets orbit the Sun moving along the
slightly elongated paths that ever change due to the changes in gravitational force between them
(which depends on their masses and distances). Such changes in the geometry of an orbit lead to
the changes in the insolation (incoming solar radiation) - quantity of heat received by any spot at
the surface of a planet. And, Milanković concluded, the overall sum of such changes must lead to

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5 Ibid, p. 817.
7 Ibid, p. 466.
the change of the thermal regime of the whole planet. So, he tried not only to prove his idea, but to calculate the exact value of the thermal change. Consequently, his theory has two parts, the astronomical and the physical one.

In the astronomical part of his work Milanković calculated the changes of the insolation of the uppermost layer of the Earth's atmosphere depending on the changes in the Earth's distance from the Sun (i.e. on the shape and magnitude of its orbit), and on the declination of Sun's rays relative to that surface unit (i.e. on orientation and inclination of rotation axis, and geographical latitude). In his investigation he incorporated three astronomical periodicities:

1. Variation in eccentricity of the Earth's orbit, from an almost exact circle to a slightly elongated shape, with the periodicity of about 100,000 years, which influences seasonal differences: when the Earth is closest to the Sun, it receives more solar radiation. If that occurs during the winter, the winter is less severe. If a hemisphere has its summer while closest to the Sun, summers are relatively warm. In addition, a more eccentric orbit will change the length of seasons in each hemisphere by changing the length of time between the vernal and autumnal equinoxes.

2. Variation in obliquity, i.e., the tilt, of the Earth's axis away from the orbital plane, from 22.1° to 24.5°; periodicity 41,000 years. At higher tilts, the seasonality at high latitudes becomes more extreme; changes in tilt have little effect in the tropics, maximum effect at the poles. For 1° of obliquity increase, the total energy received by the summer hemisphere increases by -1%. Presently, the Earth's tilt is 23.5°.

3. Precession of the equinoxes, i.e. revolution of the Earth's axis, where one revolution is completed in about 23,000 and 19,000 years. It is a rather complex phenomenon, caused by two factors: a wobble of the Earth's axis, and a turning-around of the elliptical orbit of the Earth itself. It affects the direction of the Earth's axis, not its tilt. Because the direction of the wobble is opposite to the movement of the Earth around the Sun, thus, for example, 11,000 years from now, the North Pole of the Earth will not be pointing towards the North Star, but will be pointing away by an angle of about 47°, close to the star Vega. The combined and complex wobbly motion of the Earth has the following result: the equinoxes do not keep occurring during the same day of the calendar, but slowly shift. As a result of this migration of equinoxes we have today relatively short, warm winters in the northern hemisphere and relatively long, cool summers; -11,000 years ago the opposite was true. This is the principal manifestation of the precession - the relative length of the seasons varies cyclically with time.

These three variations, superposed, constantly change the Earth's position relatively to the Sun, and, consequently, the insolation of any given spot on its surface. Milanković calculated those changes.

In the second, physical part of the theory, he had to make use of a number of physical laws in order to discover the relations between irradiation and temperature of the planets. He had to introduce various parameters for definition of the atmosphere and the ground influences on transformation of incoming radiation. Milanković here went deeply into the realm of meteorology and climatology, endowing them with a set of valuable and much needed new insights and parameters for successful use of the mathematical apparatus. First, he did not take in consideration the currents caused by the unequal heating of the atmosphere and the oceans, so the climate he had computed corresponded to the so-called solar climate. For additional insight, he determined the mean annual solar temperatures of all of the latitudes 0°-90°, their vertical structure at radiation equilibrium, and the influence of the mean content of water vapor upon it. Then, he mathematically described the exchange of heat in the ground, being aware that ground plays an
important climatic role as a reservoir of incoming solar radiation, and determined the average annual heat balance of the atmosphere.

When these, the most important problems of the theory were solved, and a solid foundation for a further work built, Milanković published his *Mathematical Theory of Heat Phenomena Produced by Solar Radiation* in 1920. It attracted the attention of climatologist Wladimir Köppen and his son-in-law, geophysicist Alfred Wegener. In 1922, they invited him to cooperate in their work *Climates of Geological Past* and to find out, using his method, the secular changes in insolation during the past 650 millennia. Köppen suggested paying special attention to secular changes in summer insolation at latitudes around 60°N, being convinced that a reduction in summer insolation at these latitudes is needed for a southward movement of the ice boundary. Another key was the insight that the insolation at high latitudes in the northern hemisphere is prevalent because most of the land that can support expanded ice accumulation belongs to this hemisphere.

With these considerations in mind, and using his method, Milanković calculated the amplitudes of secular changes which had occurred in the summer insolation at the latitudes of 55°, 60° and 65° N during the last 650,000 years, transforming his numerical results into fictitious oscillations in latitudes in order to obtain a vivid graphical picture. The quantities of heat supplied to individual latitudes by radiation during the caloric half-years were presented in canonic units in *Canon's Table XXV* (pp. 513–540). This table, with 5,600 numbers, “mathematically represent[ed] the history of insolation for the past 600 millennia — a so called Canon of insolation”8. Accordingly “with this canon one could attempt to study the phenomenon of the Ice Ages”9.

The table, transformed to a jagged line, later called 'Radiation Curve' or 'Curve of Insolation' was shown for the first time in Wegener – Köppen's book, published in 1924. One of the most important results of that publication was that Köppen in radiation diagrams recognized the evolution of then believed to have happened four glacial periods. After that, the radiation curves gained wide publicity and served as a starting point for other basic studies by climatologists and geologists. A few decades later it was realized that the four glacial periods ( Günz, Miindel, Riss, Würm) proposed by German geographers Penck and Brückner, in fact did not exist; however, the relevance of Milanković's Curve of Insolation was verified when new evidences for the cyclicity of glaciations were obtained from the records left in deep-sea sediments (Hays, Imbrie, Shackelton, 1976).

In his subsequent work Milanković extended his curve to cover the past million years, following this by a reexamination of the accuracy of ice-ages chronology by renewed computations of secular changes of astronomical elements, this time on the basis of more accurate determination of planets' masses. New calculations were not very much different from the previous ones. Subsequently he presented continuous radiation curves for the entire surface of the Earth and computed variations of insolation at eight northern and eight southern latitudes, for both the winter and summer half-years and for the top of the atmosphere as well. He showed that seasonal differences in insolation caused by the precessional cycle are of a greater significance at the lower latitudes then at the higher ones, where the influence of variations in axial obliquity is dominant.

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8 Milutin Milankovic, *Canon of Insolation of Earth and its Application to the Problem of Ice Age*, Agency for Textbooks, Belgrade 1997, p. 512. Translated into English by Israel Program for Scientific Translation under the auspices of US Department of Commerce and the National Science Foundation.

9 Ibid, p. 543.
He also deduced that the altitude of the snow line was highly dependent (correlation factor of 0.996) on the radiant energy received during the caloric summer half-year. He made tables for easy determination of the displacement of snow line at individual latitudes caused by their changes in solar insolation. Going further, he calculated the changes of the polar ice caps, which reflect a considerable part of incoming heat into outer space. He established a mathematical relationship between enlarged ice caps of the Earth and their cooling action, and showed that such diminishing of insolation represents a secondary effect, but still sufficient (when added to secular changes of insolation) to cause the great glaciations of prehistoric times in their full extent.

The most important results of Milanković's thirty years research were summarized, completed, and presented to scientific public in his capital work *Canon of Insolation and the Ice-Age Problem*, which was written in German and published in Belgrade by the Royal Serbian Academy of Science, in 1941. Later World War II events were not favorable for continual scientific work, but Milanković was not upset, because he considered his theory completed. From then on, he kept his attention on its echoes in scientific world, only occasionally delivering lectures and taking part in discussions at scientific meetings. But owing to *Canon of Insolation* it became evident that there have been multitude of ice ages during the earth history. Especially the last million years have been marked by many cycles of continental glaciation and melting. Changes in global temperature have caused massive glacial advances and retreats. That insight was confirmed definitively through the global projects CLIMAP, COHMAP and SPECMAP which mapped out the patterns of global climate change. They have demonstrated the central role of Milanković forcing, along with the response of the climate system, and made transparent that the climate system appears to act in response to insolation forcing in each Milanković cycle.

In the past several decades new cognitions launch a series of new questions which challenges Milanković's theory as well. There are geological evidences which raise questions and models which dispute it because of difficulties in reconciling theory with certain singular observations and the presence of non-orbital spectral peaks in the climate record. The detailed mechanism involved in the transformation of orbit parameter variations into climate variations are not yet known and consequently the accurate determination of the response time between astronomical forcing and climate change as well.

Nevertheless the Milanković's theory still can be tested and it is frequently confirmed by making a "simplest possible" assumption: that frequencies in the system output (orbital variations) appear linearly in the system output (climate variations). Many independent investigators appear to see clear evidence of such astronomical forcing, as well as evidence suggesting that climate system responds nonlinearly to all Milanković frequencies.

All those problems confronting the *Canon of Insolation* don’t disregard its validity as a method on which the contemporary climatology is based. It is unavoidable that Milanković saved almost discarded astronomical theory of climate and established a firm buckle, linking the exact sciences (celestial mechanics, spherical astronomy, mathematical physics) and descriptive sciences (geology, meteorology, geography, oceanology, glaciology). He set a reliable method for reconstruction and prediction of climate, which is basically still valid. "It remains, however, that the basic of all sciences involved in any theory of paleoclimates can be found in the Milanković’s *Canon*. Critically read, it will remain for ever a milestone in climate science. It is owing to the careful work by Milanković that we may expect starting to understand how the Earth system is responding to the astronomical forcing and how it might behave in the future."**10**